4.11 Solving Problems Using Quadratic Relations

What we have learned that we will be using:

- factoring and the quadratic formula leads to the roots

- finding the vertex (by factoring, partial factoring, or completing the square) gives you the optimal value

Remember that in word problems it is always important to identify the variables and sketching the parabola can be useful.

Ex.1 A hose is placed on an aerial ladder. The hose sprays water on a forest fire. The height of the water, *h*, in metres can be modelled by the relation: $h = -2.25(d - 1)^2 + 9$, where *d* is the horizontal distance, in metres, of the water from the nozzle of the hose.

a) What is the maximum height reached by the water?

∴ max height reached by the water is 9 metres

b) At what horizontal distance from the nozzle is the maximum height reached?

∴ the max height reached at a distance of 1m from the nozzle of the hose

c) What is the height of the aerial ladder? $d=0$

$=-2.25(0-1)^2+9$

$=6.75$

∴ the ladder is 6.75m high.

d) How high is the water when it is at a horizontal distance of 2 m from the nozzle? $d=2$

$=-2.25(2-1)^2+9$

$=6.75$

∴ the height of the water at $d=2$ is 6.75m.

Ex.2 A ball is thrown into the air. Its height, in metres, after *t* seconds, is $h = -4.9t^2 + 39.2t + 1.75$.

a) When does it reach maximum height?

time

$h=-4.9t^2+39.2t+1.75$

$=-4.9(t^2-8t)+1.75$

$=-4.9(t^2-8t+16-16)+1.75$

$=-4.9[(t-4)^2-16]+1.75$

$=-4.9(t-4)^2+78.4+1.75$

$=-4.9(t-4)^2+80.15$

∴ it reaches it max height after 4 secs.

$x=\frac{-b}{2a}$

$=\frac{-39.2}{2(-4.9)}$

$=4$

b) What is the maximum height?

∴ the max height is 80.15 m.

Ex.2 A ball is thrown into the air. Its height, in metres, after *t* seconds, is $h = -4.9t^2 + 39.2t + 1.75$.

c) From what height is the ball released? $t=0$

$h=-4.9(0)^2+39.2(0)+1.75$

$=1.75$

∴ the ball is released from 1.75m above the ground.

d) When does the ball hit the ground?

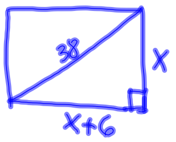
$=\frac{-(39.2)\pm\sqrt{(39.2)^2-4(-4.9)(1.75)}}{2(-4.9)}$

$x_1 = 8.044$

$x_2 = \boxed{-0.044}$ Reject Can't have negative time

∴ it hits the ground after 8 secs.

Ex.3 The size of a television screen or computer monitor is usually stated as the length of the diagonal. A screen has a 38-cm diagonal. The width of the screen is 6 cm more than the height. Find the dimensions of the screen to the nearest tenth.



$c^2 = a^2 + b^2$

$38^2 = (x+6)^2 + x^2$

$1444 = (x+6)(x+6) + x^2$

$1444 = x^2 + 12x + 36 + x^2$

$0 = 2x^2 + 12x - 1408$

a b c

$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(2)(-1408)}}{2(2)}$

$x_1 \doteq 23.70$

$\boxed{x_2 = -29.70}$ Reject cannot have a negative measurement.

∴ the the width is 29.7 cm & the height is 23.7 cm

Ex. 4 A sporting goods store sells 90 ski jackets in a season for $200 each. Each $10 decrease in price would result in five more jackets being sold.

(a) Find the number of jackets sold and the selling price to give maximum revenues.

$0 = (90 + 5x)(200 - 10x)$

or

$90 + 5x = 0$ $200 - 10x = 0$

$x = -18$ $x = 20$

$AOS = \frac{-18+20}{2}$

$\to x = 1$

Sub x=1 to solve Revenue

$= (90 + 5(1))(200 - 10(1))$

$= 18050$

∴ they need to sell 95 jacket to max revenue which is $18050

(b) What is the lowest price that would produce revenues of at least $15600?

$15600 = (90 + 5x)(200 - 10x)$

$15600 = 18000 + 1000x - 900x - 50x^2$

$0 = -50x^2 + 100x + 2400$

$= -50(x^2 - 2x - 48)$

$= -50(x-8)(x+6)$

$x_1 = x = 8$ ∴

$\boxed{x_2 = x = -6}$ reject

∴ the lowest x price would be $120 110 to produce revenue of $15600

(c) How many jackets would be sold at this price?

130

Assigned Work: p. 357 #2, 3, 5, 7, 9, 14